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SURFACE WAVE CONVERSION AT A SURFACE IMPEDANCE DISCONTINUITY OVER THE EARTH

Final Report

Principal investigator: Giorgio Franceschetti

Istituto Elettrotecnico, Università di Napoli, Italy

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Mixed-path propagation along a plane Earth is studied by applying the rigorous theory of scattering by a surface impedance discontinuity. The theory, originally available only for the normal incidence case, has been extended to the oblique incidence case. Results of the analysis have been specified to the case of surface wave incidence and to the case of a magnetic line excitation.

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SUMMARY

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KEY WORDS

Electromagnetic scattering. Mixed-path propagation. Surface waves.

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Appendixes

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1. Introduction

Groundwave propagation is one of the oldest studied topics in applied electromagnetics. Several models of the propagation medium have been considered, e.g., flat or spherical Earth, homogeneous, stratified or anisotropic ground, corrugated surfaces, single or mixed path propagation. An excellent summary of problems and modern solutions to this subject is given in [1] , which contains also a large number of references.

A canonical problem in mixed-path propagation is that of an abrupt discontinuity in ground parameters along a straight line, as depicted in Fig. 1; the ground exibits different properties, e.g., conductivity and/or permittivity, for $y > 0$ and $y < 0$. The canonical problem is the following: for a given incident plane wave, compute the field at point P. If the plane wave solution is needed to synthesize the field produced by localized sources, the angle ϕ_o should be allowed to be complex, and the incidence should not be restricted to the normal case (two-dimensional problem).

A central role in the solution to this problem is played by the surface impedance concept, i.e., by definition, the ratio of the tangential components E_t and H_t of the fields at the interface $x = 0$. For the

problem of an incident plane wave with a fixed angle ϕ_o, the use of an impedance type boundary condition is an exact approach. However, when using plane wave expansions (as in the solution of the just mentioned canonical problem), the use of an angle independent surface impedance implies an approximation to the exact boundary conditions.

A discussion about the validity of the surface impedance concept is given in [2] and, more recently, in [3] . We want also to mention a few of the excellent agreements between experimental and theoretical results using impedance type boundary conditions: see [4, 5] for the case of scattering by a half-plane with two face impedances, and [6-8] for the case of laboratory models of groundwave propagation. Additional results are listed in [1] . Accordingly, the use of impedance type boundary conditions seems quite adequate to the case at hand.

The solution to the canonical problem depicted in Fig. 1 is known essentially for the case of normal incidence [9-16] ; when oblique incidence is considered, only an approximate matching of tangential components of field at the impedance discontinuity line is provided; and restrictions on surface impedances are imposed, e.g., one of them is zero [17-18] . We note that the problem of scattering, under normal incidence, by a wedge of arbitrary angle with two different face impedances

has been rigorously solved since 1958 by Maliuzhinets [19] . When the wedge angle is made equal to π, the problems of Fig. 1 is recovered. Maliuzhinets' solution has been recently extended to the case of oblique incidence, for the half-plane case [20] , and to the case of a planar surface impedance discontinuity [21] .

Before considering the extension of Maliuzhinets' solution and its modifications appropriate to groundwave propagation problems, it is worth remembering that an alternative convenient way to the study mixed-path propagation problems is obtained by using the compensation theorem [22] . This procedure has been originally exploited by Wait [23-24] and then applied to oblique propagation of groundwaves across a coastline [25-27] . Extensive theoretical and experimental work, using essentially this procedure, is referred to in[1]. Use of the compensation theorem is very attractive inasmuch as it allows one to consider not only the simple canonical problem of Fig. 1, but, in principle, any geometry of surface discontinuity, therefore rendering the procedure very powerful from the application viewpoint. However, the resulting integral equation should be solved numerically in general, even when some simplifying approximations are made. Accordingly the rigorous solution of the canonical problem, e.g., the one depicted in

Fig. 1, is worth inasmch a physical description of the refraction phenomena at the impedance discontinuity is obtained. We also mention that mode-matching techniques have been used for solving idealized canonical problems [28] . Comparison between different theories is given in Ref. |29| .

In the rigorous solution of the problem of Fig. 1 a key role is obviously played by the two surface impedances Z^+ and Z^- or, better, by the related parameters

$$\sin\theta^{\pm} = \frac{Z^{\pm}}{\zeta \sin\beta_o}, \qquad \zeta = \sqrt{\frac{\mu_o}{\varepsilon_o}} \qquad (1.\ 1)$$

where β_o is related to the ablique incidence angle (see Fig. 2 and 1. 1) and the ± signs refer to $y > 0$ (plus) and $y < 0$ (minus) respectively (when confusion does not arise, we omit the superscripts).

For groundwave propagation

$$Z = \sqrt{\frac{\mu}{\varepsilon_o\,\varepsilon_r - \frac{\sigma}{i\omega}}} \simeq \sqrt{\frac{-i\omega\mu_o}{\sigma}} = \frac{1}{\sigma\,\delta}(1 - i) \qquad (1.\ 2)$$

wherein ε_r is the relative permeability, σ the conductivity and δ the skin dept; a time dependance $\exp(-i\omega t)$ has been assumed; and the approximation requires:

$$\omega\varepsilon_o\varepsilon_r \ll \sigma \qquad (1.\ 3)$$

Inequality (1. 3) is satisfied up to frequencies $f \simeq 10$kHz for a very dry soil ($\sigma \simeq 10^{-4}$ siemens/m; $\varepsilon_r \simeq 15$) and up $f \simeq 1$ MHz for normal soil ($\sigma \simeq 10^{-2}$ siemens/m, $\varepsilon_r \simeq 15$); over the sea ($\sigma \simeq 5$ siemens/m, $\varepsilon_r \simeq 80$) the inequality is valid in all the frequency range of groundwave propagation. In any case, the (complex) angle θ defined by (1. 1) is always small, and usually very small. We are consistently using this last assumption in all the body of this report. Accordingly, we take (for $\beta = 90°$)

$$\theta \simeq \frac{Z}{\zeta} \simeq \frac{1}{\sigma\delta\zeta}(1-i), \qquad [\theta] \ll 1 \qquad (1.\ 4)$$

It is also necessary to introduce another characteristic angle, related to the normalized surface admittance, hence:

$$\sin \gamma^{\pm} = \frac{\zeta}{Z^{\pm} \sin\beta_o} \qquad (1.\ 5)$$

The (complex) angle γ can be easily computed under the approximation (1. 2), hence (for $\beta = 90°$)

$$\gamma \simeq \frac{\pi}{4} + i \ln \frac{\sigma\delta\zeta}{\sqrt{2}} \simeq \frac{\pi}{4} - i \ln [\theta]; \qquad (1.\ 6)$$

$$[\gamma] > 1$$

In passing, we note that the underlying assumptions leading to (1. 4 and 6) are also sufficient to validate the use of the surface impedance concept for non plane wave excitation.

It has been first recognized by Senior [17-18] that propagation of waves along a mixed-path, as depicted in Fig. 1, consist assentially of a scattering problem, although it is usually referred to as a refraction one, in the frame of surface propagation (for istance, costal refraction). This is clearly shown in the Maluizhinets' theory, wherein the total field is decomposed in its various components: the incident one (a plane wave), the two reflected waves on the two surfaces $y < 0$ and $y > 0$, the two surface waves along $y < 0$ and $y > 0$, and the (cylindrical) wave scattered by the line impedance discontinuity $y = 0$. Particularly important are the surface waves, since they are characteristic of the surface and, therefore, less sensitive to the approximation of the model, e.g., undulation of the surface or Earth's curvature.

In the two-dimensional case of Fig. 1 (z-independent fields) an H-polarized surface field (the only of interest in radiopropagation along the Earth's surface) is characterized by a propagation factor(the wave progresses along - y);

$$\exp(-ikx \sin\theta - iky \cos\theta) = \exp\left[-ik\rho \cos\left(\phi - \frac{\pi}{2} + \theta\right)\right] \qquad (1.\ 7)$$

wherein $k^2 = \omega^2 \varepsilon_o \mu_o$ and is the free-space propagation constant. According

the incident angle is

$$\phi_o = \frac{\pi}{2} - \theta \tag{1. 8}$$

For the three-dimensional case of Fig. 3, wherein the field propagates with a (real) angle β with respect to the z-axis, the appropriate propagation factor is obtained from (1. 7) by using the rotation of the coordinates

$$y = y \sin\beta - z \cos\beta \tag{1. 9}$$

Accordingly, we get the new propagation factor

$$\begin{aligned} &\exp(-ikx \sin\theta - iky \sin\beta \cos\theta + ikz \cos\beta \cos\theta) = \\ &= \exp\left[-ik\rho\cos(\phi - \phi_o)\right] \exp(ikz \cos\beta_o) \end{aligned} \tag{1. 10}$$

Therefore, the incidence angles ϕ_o, β_o are given by

$$\left\{\begin{aligned} \cos\beta_o &= \cos\theta\cos\beta \\ \cos\phi_o &= \frac{\sin\theta}{\sqrt{1 - \cos^2\theta\cos^2\beta}} \\ \sin\phi_o &= \frac{\cos\theta\ \sin\beta}{\sqrt{1 - \cos^2\theta\cos^2\beta}} \end{aligned}\right. \tag{1. 11}$$

Accordingly, to the first order in θ

$$\left\{\begin{aligned} \beta_o &\simeq \beta \\ \phi_o &\simeq \frac{\pi}{2} - \theta \end{aligned}\right. \tag{1. 12}$$

The simple source model for exciting surface waves over an impedance

plane is an infinite magnetic line of intensity I_m, as depicted in Fig. 3. The free space magnetic field produced by the line is z-polarized and given by

$$H = \frac{k\ I_m}{4\ \zeta} H_o^{(1)}(kd) \qquad (1.\ 13)$$

wherein $H_o^{(1)}(x)$ is the Hankel function of the first kind. Let us use the integral representation of the Hankel function:

$$H_o^{(1)}(kd) = \frac{1}{\pi} \int_S \exp\left[ikd \cos(\alpha-\eta)\right] d\,\alpha =$$

$$= \frac{1}{\pi} \int_S \exp\left[ik\rho_o \cos(\alpha-\eta_o)\right] \exp\left[-ik\rho \cos(\phi-\frac{\pi}{2}+\alpha)\right] d\alpha \qquad (1.\ 14)$$

wherein S in the Sommerfield contour of integration depicted in Fig. 4. Inspection of (1. 14 and 13) shows that the line source is synthesized as superposition of plane waves of (variable) incidence angle

$$\phi_o = \frac{\pi}{2} - \alpha \qquad (1.\ 15)$$

and of spectral intensity

$$\exp\left[ik\rho \cos(\alpha-\eta_o)\right] \qquad (1.\ 16)$$

Accordingly, superposition of the plane wave solution weighted by the factor (1. 16) allows the computation of the field produced by the line source.

As a last remark in this introductory section, let us state that the Sommerfeld contour of integration S, after deformation into the steepest descent path, transforms in the real axis, from $-\infty$ up to $+\infty$, upon use of the variable change

$$\sqrt{2}\,\exp(i\,\frac{\pi}{4})\,\sin\frac{\alpha-\eta}{2} = t \qquad (1.17)$$

2. Mathematical background

In the Muliuzhinets' theory of scattering by the configuration depicted in Fig. 1, a central role in played by the Maliuzhinets' functions [19]

$$\Psi_h = \Psi_h(\alpha\,,\,\Theta)\;;\;\Psi_e = \Psi_e(\alpha,\gamma) \qquad (2.1)$$

The two functions are identical, the second being given by the first one upon replacement of γ with Θ. Accordingly, only the function Ψ_e is presented hereafter, and the subscript and Θ dependance is dropped. We have:

$$\Psi(\alpha) = N(\alpha+\pi-\overset{+}{\Theta})\,N(\alpha+\overset{+}{\Theta})\,N(\alpha-\pi+\Theta^{-})\,N(\alpha-\overset{-}{\Theta}) \qquad (2.2)$$

or, equivalently:

$$\Psi(\alpha) = N^2(\frac{\alpha}{2})\frac{N(\alpha+\overset{+}{\Theta})\,N(\alpha-\overset{-}{\Theta})}{N(\alpha-\overset{+}{\Theta})\,N(\alpha+\overset{-}{\Theta})}\cos\frac{\alpha-\overset{+}{\Theta}+\frac{\pi}{2}}{2}\cos\frac{\alpha+\overset{-}{\Theta}-\frac{\pi}{2}}{2} \qquad (2.3)$$

The function $N(\nu)$ is given by

$$N(\nu) = \exp\left[\frac{1}{4\pi}\int_0^{\nu} \frac{2t - \pi\sin t}{\cos t}\, dt\right] \tag{2. 4}$$

and is a particular determination of a most general class of function introduced by Maliuzhinets [19] with reference to the problem of impedance wedge scattering. The function $N(\nu)$ is easy to compute numerically and, in any case, it is simply related to a tabulated function [20, 30] . Its main properties are hereafter summarized:

$$N(\nu) = N(-\nu) \tag{2. 5}$$

$$N(\nu + \frac{\pi}{2})\, N(\nu - \frac{\pi}{2}) = N^2(\frac{\pi}{2}) \cos \frac{\nu}{2} \tag{2. 6}$$

$$\lim_{|\nu''| \to \infty} N(\nu) \simeq O\left[\exp \frac{[\nu'']}{4}\right] \quad ; \quad \nu = \nu' + i\,\nu'' \tag{2. 7}$$

The location of poles of $\Psi(\alpha)$ within the strip

$$-\frac{3\pi}{2} < \alpha' < \frac{3\pi}{2}$$

is of interest. From (2. 4) it is noted that

$$\int_0^{\alpha} \frac{2t - \pi\sin t}{\cos t}\, dt \tag{2. 8}$$

is finite for $\alpha=\pi/2$, diverges toward negative values for

$\alpha = 3\pi/2$ and toward positive values for $\alpha = 5\pi/2$. Since the divergence of (2. 8) is of logarithmic type, the function $N(\alpha)$ has a simple pole at $\alpha = \pm 5\pi/2$. Accordingly, the pole of $\Psi(\alpha)$ in the strip of interest are, from (2. 2):

$$\alpha^{\pm} = \pm\left(\frac{3\pi}{2} + \theta^{\pm}\right) \qquad (2.\ 9)$$

An appropriate use of relations (2.2 - 3 - 5 - 6) may avoid the necessity of effective computation of the function $\Psi(\alpha)$. We quote here after those cases which are relevant to subsequent analysis.

Use of (2. 3 - 5 - 6) gives

$$\Psi_h\left(\frac{\pi}{2} + \tau\right) = \frac{N^2\left(\frac{\pi}{2}\right)N\left(\frac{\pi}{2} + \theta^+ + \tau\right)N\left(\frac{\pi}{2} - \theta^- - \tau\right)N\left(\frac{\pi}{2} - \theta^- - \tau\right)}{N\left(\frac{\pi}{2} - \theta^+ + \tau\right)} \cdot$$

$$\cdot \sin\frac{\theta^+ - \tau}{2} \qquad (2.\ 10)$$

If, as assumed, θ is small and τ is prescribed to assume only small values, then

$$\Psi_h\left(\frac{\pi}{2} + \tau\right) \simeq N^4\left(\frac{\pi}{2}\right)\sin\frac{\theta^+ - \tau}{2} \qquad (2.\ 11)$$

Similarly

$$\Psi_h\left(-\frac{3\pi}{2} \pm \tau\right) = N^4\left(\frac{\pi}{2}\right)\frac{N\left(\frac{\pi}{2} - \tau + \theta^+\right)N\left(\frac{\pi}{2} - \tau - \theta^-\right)}{N\left(\frac{\pi}{2} - \tau - \theta^+\right)N\left(\frac{\pi}{2} - \tau + \theta^-\right)} \cdot$$

$$\cdot\ \frac{\sin\frac{\theta^{+}+\tau}{2}\,\sin\frac{\theta^{-}-\tau}{2}}{\sin\frac{\theta^{-}+\tau}{2}}\,\cos\frac{\theta^{-}+\tau}{2}\approx$$

$$\approx N^{4}\left(\frac{\pi}{2}\right)\left[\sin\frac{\theta^{+}+\tau}{2}\,\sin\frac{\theta^{-}-\tau}{2}\Big/\sin\frac{\theta^{-}+\tau}{2}\right] \qquad (2.\ 12)$$

under the same previous assumptions.

To complete this section let us quote an integral which is systematically used in the body of this Report and which can be evaluated properly accomodating results of Ref.[31]:

$$I(\Omega)=\sqrt{2}\,\exp\left(i\frac{\pi}{4}+i\,\Omega\right)\sin\frac{\theta}{2}\int_{-\infty}^{+\infty}\frac{\exp(-\,\Omega t^{2})}{t^{2}-2i\,\sin^{2}\frac{\theta}{2}}\,dt=$$

$$=\pi i\,\exp(i\,\Omega\cos\theta)\,\frac{F\left(\sqrt{2\Omega}\,\sin\frac{\theta}{2}\right)}{F\,(0)} \qquad (2.\ 13)$$

wherein $F(x)$ is the Fresnel integral

$$F(x)=\int_{x}^{\infty}\exp(i\,t^{2})dt \qquad (2.\ 14)$$

3. Mixed-path propagation. Formal solution for plane wave oblique incidence

Let us consider the (three-dimensional) problem of plane wave scattering by the mixed-path configuration depicted in Fig. 2. Any component F the incident plane wave is represented

by

$$G^i(\rho, \phi, z) = G_o^i \exp(ikz \cos\beta_o)\exp\left[-ik\rho \sin\beta_o \cos(\phi-\phi_o)\right] \quad (3.1)$$

Without loss of generality we take $0 < \mathrm{Re}\left[\beta_o\right] \leq \frac{\pi}{2}$ and drop the z-dependence of the field components.

Boundary conditions are expressed as

$$\underline{E} - \underline{i}_x\, \underline{i}_x \cdot \underline{E} = Z^{\pm} \underline{i}_x \times \underline{H} \quad \text{at } \phi = \pm\frac{\pi}{2} \quad (3.2)$$

or, equivalently,

$$\underline{H} - \underline{i}_x \underline{i}_x\ \underline{H} = \frac{1}{Z^{\pm}} \underline{E} \times \underline{i}_x \quad \text{at } \phi = \pm\frac{\pi}{2} \quad (3.3)$$

By taking the divergence of (3.2 and 3) and using Maxwell's equations, we get

$$\frac{1}{\rho}\frac{\delta E_x}{\delta \phi} \mp ik \sin\Theta^{\pm} E_x = 0 \quad \text{at } \phi = \pm\frac{\pi}{2} \quad (3.4)$$

$$\frac{1}{\rho}\frac{\delta H_x}{\delta \phi} \mp ik \sin\gamma^{\pm} H_x = 0 \quad \text{at } \phi = \pm\frac{\pi}{2} \quad (3.5)$$

respectively.

Let us now expand any component G of the total field as a superposition of plane waves, hence:

$$G(\rho,\phi) = \frac{1}{2\pi i}\int_{\Gamma} \hat{G}(\alpha+\phi)\exp(-ik\rho \sin\beta_o \cos\alpha)\,d\alpha \quad (3.6)$$

where Γ is the two-branch contour of integration depicted in Fig. 5.

Forcing boundary conditions (3.4 and 5) in (3.6) we

get, upon integration by parts

$$\int_{\Gamma} (\sin\alpha \pm \sin\theta^{\pm})\hat{E}_x(\alpha \pm \frac{\pi}{2})\exp(-ik\rho \sin\beta_o \cos\alpha)d\alpha = 0 \quad (3.\ 7)$$

$$\int_{\Gamma} (\sin\alpha \pm \sin\gamma^{\pm})\hat{H}_x(\alpha \pm \frac{\pi}{2})\exp(-ik\rho \sin\beta_o \cos\alpha)d\alpha = 0 \quad (3.\ 8)$$

Solution to equations of type (3. 7 - 8) is given in [19] under the assumtion that a simple pole does exit at $\phi = \phi_o$:

$$\hat{G}(\alpha) = \left[\frac{G_o^i \cos\phi_o}{\sin\alpha - \sin\phi_o} + \sum_{o}^{\infty} {}_n G_n \sin^n\alpha\right]\frac{\Psi(\alpha)}{\Psi(\phi_o)} \quad (3.\ 9)$$

wherein G_o^i is clearly the residue at $\phi = \phi_o$, G_n are arbitrary constants and the function $\Psi(\alpha)$ is the Maliuznets' function introduced in Sect. 2.

In order to determine these unknown constants G_n let us express the spectral z-components of the field $(\hat{E}_z, \hat{H}_z)$ in terms of the x-components $(\hat{E}_x, \hat{H}_x)$. We have (see Appendix A)

$$\hat{E}_z(\alpha) = \frac{\cos\alpha\cos\beta\hat{E}_x(\alpha) - \sin\alpha\zeta\hat{H}_x(\alpha)}{1-\cos^2\alpha\sin^2\beta_o}\sin\beta_o \quad (3.\ 10)$$

$$\zeta\hat{H}_z(\alpha) = \frac{\cos\alpha\ \cos\beta\zeta\hat{H}_x(\alpha) - \sin\alpha\hat{E}_x(\alpha)}{1-\cos^2\alpha\sin^2\beta_o}\sin\beta_o \quad (3.\ 11)$$

As discussed in [32], the value of any field components G for $\rho \to 0$ is related to the asymptotic behaviour of the corresponding spectral component $\hat{G}$ as $[\alpha''] \to \infty$. In particular, $\hat{G}$ should

be bounded if G is required to be regular as it happens for E_z and H_z.

Substituting the expansion (3. 9) in (3. 10 and 11) and using (2. 7), it is easy to convince ourselves that all G_n must be equal to zero with the exception of G_o. The conclusion is that the appropriate expansion for $\hat{E}_x$, $\hat{H}_s$ is the following one:

$$\hat{E}_x(\alpha) = \left[\frac{E_{o_x}\cos\phi_o}{\sin\alpha - \sin\phi_o} + E_h\right] \frac{\Psi_h(\alpha)}{\Psi_h(\phi_o)} \qquad (3.\ 12)$$

$$\hat{H}_x(\alpha) = \left[\frac{H_{o_x}\cos\ \phi_o}{\sin\ \alpha - \sin\phi_o} + H_e\right] \frac{\Psi_e(\alpha)}{\Psi_e(\phi_o)} \qquad (3.\ 13)$$

wherein E_{o_x}, H_{o_x} are the x-components of the incident field and E_h, H_e two still unknown constants.

In order to evaluate those constants, we note that the spectral components $\hat{E}_z$, $\hat{H}_z$ as well as $\hat{E}_y$, $\hat{H}_y$ exhibit simple poles in the integrand (3. 6) at values of α given by

$$\cos(\alpha+\phi) = \pm \frac{1}{\sin\beta_o} \qquad (3.\ 14)$$

When the integral (3. 6) is evaluated in terms of its singularities, these poles generate plane waves whose y-dependence is of the type:

$$\exp(\pm ky \cos\beta_o) \qquad (3.\ 15)$$

therefore diverging for positive or negative values of y. In order to have a field solution which is everywhere bounded, the singularities (3. 14) must be compensated forcing the numeratores in (3. 10 - 11) to be zero at values of α given by (3.14). This can be obtained by letting

$$\hat{E}_x(\alpha_p^{\pm}) - i\zeta\, \hat{H}_x(\alpha_p^{\pm}) = 0 \qquad (3.\ 16)$$

wherein

$$\cos\alpha_p^{\pm} = \frac{1}{\sin\beta_o} \qquad (3.\ 17)$$

and, consequently:

$$\sin\alpha_p^{\pm} = \pm i \cot g\beta_o \qquad (3.\ 18)$$

It can be checked by ispection that relation (3. 16) forces also $\hat{E}_y$, $\hat{H}_y$ (see A. 10 - 11) to be finite at $\alpha = \alpha_p^{\pm}$.

Condition (3. 16) provides a system of two equations in the two unknown E_h, H_e. Once these are evaluated, the spectral components of the field are completely known.

4. Spectral field singularities

For a discussion about the properties of the spectral

representation (3. 6) it is convinient to deform the original integration contour Γ into the two-branch steepest descent path C (see Fig. 5), so that

$$G(\rho,\phi) = \frac{1}{2\pi i}\int_C \hat{G}(\alpha+\phi)\exp(-ik\rho\,\sin\beta_o\,\cos\alpha)d\alpha + \tag{4. 1}$$
$$+\Sigma\left[\text{Residues of } \hat{G}(\alpha+\phi) \text{ in } (\Gamma-C)\right]$$

The function $\hat{G}(\alpha+\phi)$ exhibits singularities coincident with those (3. 9). Poles of the first factor in (3. 9) do accour at:

$$\alpha_n = -\phi+(-)^n\phi_o+n\pi; \quad n = 0\ ; \pm1; \ldots\ldots \tag{4. 2}$$

Poles of the second factor do accour at

$$\alpha_\pm=\pm\left(\frac{3\pi}{2} + \theta^\pm\right) + 2n\pi \tag{4. 3}$$

If those poles are located inside (Γ-C), the corresponding residues generate plane-wave fields.

In particular, the pole α_o corresponds to the incident wave; the two poles $\alpha_{\pm1}$ to the (geometrically) reflected waves upon the two half-planes $\phi=\pm\pi/2$ respectively; and the poles $\alpha_\pm$ to surface waves upon the same half-planes.

Furthermore, the integrand in (4. 1) exhibits saddle points at $\alpha = -\phi\pm\pi$. In the asymptotic evaluation of the field, these saddle points generate cylindrical waves from the discontinuity line $\rho = 0$

5. Surface wave incidence

As stated in Sect. 1, an important problem is the computation of the surface wave conversion at the surface impedance discontinuity for a mixed-path plane Earth propagation. Accordingly, we are taking as incident wave a TM (with respect to x) surface wave (see Fig. 2) of field x-components (see 1. 10 and 11):

$$\begin{cases} E_x^i = E_{o_x} \exp(ikz\cos\beta_o)\exp\left[-ik\rho\sin\beta_o\cos(\phi-\phi_o)\right] \\ H_x^i = 0 \end{cases} \tag{5. 1}$$

wherein, to the first order in θ_h^+,

$$\beta_o \simeq \beta \quad ; \quad \phi_o \simeq \frac{\pi}{2} - \theta^+ \tag{5. 2}$$

The spectral x-components of the total field are readily obtained from (3. 12 and 13), hence:

$$\hat{E}_x(\alpha) = \left[\frac{E_{o_x}\cos\phi_o}{\sin\alpha - \sin\phi_o} + E_h\right]\frac{\Psi_h(\alpha)}{\Psi_h(\phi_o)} \tag{5. 3}$$

$$\hat{H}_x(\alpha) = H_e \frac{\Psi_e(\alpha)}{\Psi_e(\phi_o)} \tag{5. 4}$$

The fields for $\phi = -\pi/2$, i.e., on the Earth's surface $y < 0$ are of interest. The three poles (see 4. 2 and 3)

$$\begin{cases} \alpha_{-1} = -\pi+\theta^+ \\ \alpha_{-2} = -\pi-\theta^+ \\ \alpha_- = -\pi-\theta^- \end{cases} \tag{5. 5}$$

are located near the saddle point $\alpha = -\pi$ as far as $\hat{E}_x(\alpha)$ in concerned, while $\hat{H}_x(\alpha)$ is regular in the same neighbour. The two poles

$$\begin{cases} \alpha_o = \pi - \Theta^+ \\ \alpha_1 = \pi + \Theta^+ \end{cases} \qquad (5.\ 6)$$

are located near by the saddle point $\alpha=\pi$, again as far as $\hat{E}_x(\alpha)$ is concerned, while $\hat{H}_x(\alpha)$ is again regular in the same neighbour. Since the surface wave transmitted in $y<0$ is essentialy the contribution of the pole α_-, it follows that this wave is still TM polarized, as it would be expected, since $\hat{H}_x(\alpha)$ provides no contribution to it. For the computation of this transmitted surface field, first we must evaluate the constant E_h in (5. 3). This can be readily accomplished by forcing the condition (3. 16), hence:

$$E_h = \frac{E_{o_x} \cos\phi_o \sin^2\beta_o}{1 - \sin^2\beta_o \cos^2\phi_o} \Bigg] \sin\phi_o + i \operatorname{cotg}\beta_o \qquad (5.\ 7)$$

$$\frac{\Psi_h(\alpha_p^+)\Psi_e(\alpha_p^-) + \Psi_h(\alpha_p^-)\Psi_e(\alpha_p^+)}{\Psi_h(\alpha_p^+)\,\Psi_e(\alpha_p^-) - \Psi_h(\alpha_p^-)\Psi_e(\alpha_p^+)} \Bigg]$$

Now, in the asymptotic evaluation of the integral in (4. 1) for large $k\rho \sin\beta_o$, only values of the integrand near the saddle points $\alpha = \pm\pi$ are of interest, so that a series expansion of $\hat{E}_x(\alpha - \pi/2)$ near those two points is appropriate. In particular, we get from (2. 11 and 12)

near $\alpha = -\pi$:

$$\hat{E}_x(-\pi+\tau-\frac{\pi}{2}) \simeq \left[\frac{E_{o_x}\sin\theta^+}{\cos\tau-\cos\theta^+} + E_h\right] \frac{\sin\frac{\theta^+ +\tau}{2}\sin\frac{\theta^- -\tau}{2}}{\sin\theta^+\sin\frac{\theta^- +\tau}{2}} \qquad (5.\ 8)$$

A great care should now be taken in the asymptotic evaluation of the field; due to the clustering of two poles nearby the saddle point. The evaluation can be accomplished either by using the Bleistein method [30] or simply by extracting the singularity near the saddle point [31]. These two methods are equivalent to the first order and the latter is most convinient to our purposes since only the term singular at $\alpha=\alpha_-$ is here of interest. This term, $\hat{E}'_x(\tau)$, is given by:

$$\hat{E}'_x(\tau) = \lim_{\tau \to \theta} \sin\frac{\theta^- +\tau}{2}\,\hat{E}_x(\tau) = \left[\frac{E_{o_x}\sin\theta^+}{2(\sin^2\frac{\theta^-}{2} - \sin^2\frac{\theta^+}{2})} + E_h\right] \cdot$$

$$\cdot \frac{\sin\theta^-}{\sin\theta^+} \quad \frac{\sin\frac{\theta^+ -\theta^-}{2}}{\sin\frac{\theta^- +\tau}{2}} \qquad (5.\ 9)$$

Wherein the integration contour C in depicted in Fig. 6. A detained evaluation of the constant E_h in terms of the system parameters θ^+, θ^-, β_o is given in Appendix B. If not only θ is small, but also

$$\left[\theta \operatorname{cotg}\frac{\beta_o}{2}\right] \ll 1 \qquad (5.\ 10)$$

and (1. 6) is valid, then E_h is of order $1/\theta$ while the β_o-independent term in (5. 9) is of order $1/\theta^2$. Accordingly:

$$\hat{E}'_x(\alpha) \simeq \frac{2 \sin \theta^-}{\sin \theta^+ + \sin \theta^-} E_{o_x} \frac{1}{2(\sin \frac{\tau}{2} + \sin \frac{\theta^-}{2})} \tag{5. 11}$$

$$= \bar{T} E_{o_x} \frac{1}{2(\sin \frac{\tau}{2} + \sin \frac{\theta^-}{2})}$$

wherein T is a transmission coefficient.

The transmitted surface field upon the Earth is given by:

$$E^s_x(\rho, -\frac{\pi}{2}) = E^s_x (y) = \frac{T E_{o_x}}{4\pi i} \int_{C_o} \frac{\exp(ik\rho \sin\beta_o \cos \tau)d\tau}{\sin \frac{\tau}{2} \cos \frac{\theta^-}{2} + \sin \frac{\theta^-}{2} \cos \frac{\tau}{2}} =$$

$$= - \frac{T E_{o_x}}{4\pi i} \sin \frac{\theta^-}{2} \int_{C_o} \frac{\cos \frac{\tau}{2} \exp (ik\rho \sin \beta_o \cos \tau)}{\sin^2 \frac{\tau}{2} - \sin^2 \frac{\theta^-}{2}} d\tau \tag{5. 12}$$

wherein C_o is the integration contour depicted in Fig. 6 and the odd part of the integrand has been disregarded.

By using the change of variable (1. 17) with $\eta = 0$ the integral (5. 12) is transformed in the canonical form (2. 13) and we get:

$$E_x^s(y) = \frac{T\,E_{o_x}}{2} \exp\,(-iky \sin\beta_o \cos\theta^-) \cdot \frac{F\left[\sqrt{2k\rho\sin\beta_o}\,\sin\frac{\theta^-}{2}\right]}{F(0)} =$$

$$= \frac{T\,E_{o_x}}{2} \exp\,(-iky \sin\beta_o \cos\theta^-) \tag{5. 13}$$

if

$$2\beta\rho \sin\beta_o \sin^2\frac{\theta^-}{2} \ll 1 \tag{5. 14}$$

The result (5. 13) is interesting inasmuch as it shows that, at large distances from the surface discontinuity, the surface field is transmitted according to the ususal tranmission coefficient of the refraction theory. Furthermore, the two components of the propagation constant are given by:

$$k_z = k \cos\beta_o = k \cos\beta \cos\theta^+ \tag{5. 15}$$

$$k_y = k \sin\beta_o \cos\theta^- \simeq k \sin\beta_o \cos\theta^+ \left(1 + \frac{\sin^2\theta^+ - \sin^2\theta^-}{2}\right)$$

Eqs. (5. 15) show that the change in the phase front direction is negligible when the assumption (1. 3) is made. However, expression (5. 15), at variance of that for T, is valid also when (1. 3) is relaxed, since only the angle θ^+ is small.

6. Excitation of the surface made by a magnetic line source over the Earth

In the preceeding section it was shown that, in the case of highly conducting surfaces, propagation of the surface wave is poorly affected by the incidence angle.

This result suggests to examine the excitation of the surface wave in the two-dimensional case, i.e., when the source is taken equal to a magnetic line source parallel to the plane x = 0, as depicted in Fig. 3, with $\eta_o << 1$. From the analysis of Sect. 1 -see (1. 13 and 16) - we get the formal expression for the z-component of the total magnetic field along the surface $x = 0$, $y \leq 0$, hence

$$H_z(\rho, -\frac{\pi}{2}) = \frac{k\, I_m}{4\, \zeta} \frac{1}{\pi} \int_S G(\alpha) \exp\left[ik\, \rho_o \cos(\alpha - \eta_o)\right] d\alpha \tag{6. 1}$$

wherein S is the integration contour represented in Fig. 4 and $G(\alpha)$ is given by (3. 6 and 9) with $\beta_o = 90°$ and $G_o^i = 1$, hence:

$$G(\alpha) = \frac{1}{2\pi i} \int \frac{\cos(\frac{\pi}{2} - \alpha)}{\sin(\xi - \frac{\pi}{2}) - \sin(\frac{\pi}{2} - \alpha)} \cdot \frac{\Psi_h(\xi - \frac{\pi}{2})}{\Psi_h(\frac{\pi}{2} - \alpha)} \exp(-ik\rho \cos\xi) d\xi \tag{6. 2}$$

The integration contour is represented in Fig. 7 and can be changed in the alternative contour C provided that the residues of the integrand inside $\Gamma - C$ are taken into account.

From result of Sect. 4, it follows that the pole location is that depicted in the same Fig. 7. Since α is prescribed to run along the curve S of Fig. 4, it follows that the two poles ξ_o and ξ_{-1} are always inside the closed contour $\Gamma - 0$, for $\eta_o > 0$. The corresponding residues are given by:

$$\left[1 - \frac{\Psi_h(-\frac{3\pi}{2}+\alpha)}{\Psi_h(\frac{\pi}{2}-\alpha)} \right] \exp(ik\rho\cos\alpha) \simeq \exp(ik\rho\cos\alpha) \left[1 - \frac{\sin\frac{\bar{\theta}-\alpha}{2}}{\sin\frac{\bar{\theta}+\alpha}{2}} \right] \simeq$$

$$\simeq \frac{2\sin\alpha}{\sin\alpha + \sin\bar{\theta}} \exp(ik\rho\cos\alpha) \qquad (6.\ 3)$$

since, in the asymptotic evaluation of the integral (6. 1) for $k\rho_o$ large, only values of $\alpha = \eta_o \ll 1$ are of interest. Note that we used result (2. 11 - 12) in order to obtain the final expression (6. 3).

The integral (6. 2) along the two steepest descent paths C must now be computed. If :

$$\sqrt{2 k \rho}\ \sin \frac{\eta_o}{2} \ll 1 \qquad (6.\ 4)$$

the position of the poles $\xi_o, \xi_1, \xi_{-1}, \xi_{-2}$, is sufficiently remote from the integration path to produce any significant change in the conventional asymptotic evaluation. From another viewpoint, decomposition of the integrand (6. 2) in simple fractions and transformation via

(1. 17) in the canoni:al form (2. 13) leads to terms that are inversely propotional to (6. 4), and therefore dominated by (6. 3). This is not true for the pole ξ_-. Expanding the integrand (6. 2) nearby $\xi = -\pi$, be have, for α small,

$$\frac{\cos(\frac{\pi}{2} - \alpha)}{\sin(\xi - \frac{\pi}{2}) - \sin(\frac{\pi}{2} - \alpha)} \quad \frac{\Psi_h(\xi - \frac{\pi}{2})}{\Psi_h(\frac{\pi}{2} - \alpha)} \simeq$$

$$\simeq \frac{\sin\alpha}{\cos\tau - \cos\alpha} \quad \frac{\sin\frac{\theta^+ + \tau}{2} \; \sin\frac{\theta^- - \tau}{2}}{\sin\frac{\theta^+ + \alpha}{2} \; \sin\frac{\theta^- + \tau}{2}}, \quad \xi = -\pi + \tau \qquad (6.\ 5)$$

Extracting the singularity at $\tau = -\theta^-$ and evaluating the integral as in Sect. 5, we get the term

$$\frac{1}{2} \frac{\sin\alpha \sin\theta^- \sin\frac{\theta^+ - \theta^-}{2}}{(\sin^2\frac{\alpha}{2} - \sin^2\frac{\theta^-}{2}) \sin\frac{\theta^+ + \alpha}{2}} \cdot \exp(ik\rho\cos\theta^-) \qquad (6.\ 6)$$

which again dominates the remaining part of the asymptotic evaluation if, as we have assumed,

$$k\rho \sin^2\theta^- \ll 1 \qquad (6.\ 7)$$

The conclusion is that we can take as $G(\alpha)$ the sum of (6. 3) and (6. 6). For the term (6. 3) we have

$$\frac{1}{\pi}\int_S \frac{2\sin\alpha}{\sin\alpha+\sin\theta^-}\exp(ik\rho\cos\alpha)\exp[ik\rho_c\cos(\alpha-\eta_\theta)]\,d\alpha =$$

$$\frac{1}{\pi}\int_S \frac{2\sin\alpha}{\sin\alpha+\sin\theta^-}\exp[ik\,d\cos(\alpha-\eta)]\,d\alpha \qquad (6.\ 8)$$

upon comparison with (1. 14) and when the point $P(x, y)$ of Fig. 3 lies on the surface $x = 0$.

By using the same procedure as for the integral (5. 12) we get for the first term H_{z_1} of the magentic field for $y < 0$

$$H_{z_1}(\rho, -\frac{\pi}{2}) = \frac{k\,I_m}{4\,\zeta}\quad 2\sqrt{\frac{2}{\pi\,k\,d}}\exp(-i\,\frac{\pi}{4} + i\,k\,d) + \qquad (6.\ 9)$$

$$+\ 4i\sin\frac{\theta^-}{2}\quad \exp\left[ikd\cos(\theta^- + \eta)\right]\frac{F(\sqrt{2kd}\sin\frac{\theta^- + \eta}{2})}{F(0)}$$

When:

$$\sqrt{2k\,d}\,\sin\frac{\theta^- + \eta}{2} >> 1 \qquad (6.\ 10)$$

asymptotic expansion of the solution gives

$$H_{z_2}(\rho, -\frac{\pi}{2}) \approx \frac{k I_m}{4 \zeta} \frac{2 \sin \eta}{\sin \eta + \sin \theta^-} \sqrt{\frac{2}{\pi k d}} \exp(-i \frac{\pi}{4} + ikd) \qquad (6.\ 11)$$

i.e., the cylindrical wave radiated by the source and its *image* with respect to the plane $y = 0$.

Similar results we have for the contribution $H_{z_2}(\rho, -\frac{\pi}{2})$ due to the second term (6. 6) upon decomposition in simple fraction. Computations are straightforward.

7. Conclusions

We presented a rigorous theory of propagation along a mixed-path plane Earth. The theory is based upon the exact solution given by Maliuzhinets for the normal incidence, properly extended to the oblique incidence case. Maliuzhinets' solution is given in terms of a (relatively) complicated function.

We have evaluated this function in closed form for all cases of practical interest.

First the case of an incident surface wave has been considered. For highly conductive surfaces, it was shown that the transmission coefficient is real and practically independent from the incidence angle. This is not the case when at least one of the surfaces is poorly conducting. Although not explicitly examined, the solution

gives, in this last case a (complex) angle depending trasmission coefficient.

The direction of propagation of the phase front is not appreciably changed, for the case of highly conducting surfaces, far from the impedance discountinuity. Again, this is not true when at least one of the surfaces is poorly conducting.

The case of source excitation can be considered by synthesizing the field from the plane wave solution. Examination of the solution confirms previous results.

APPENDIX A

Relations between spectral components of the field

For the spectral components of the field we have:

$$\underline{k} \times \hat{\underline{E}} = \omega\mu \, \hat{\underline{H}} \qquad (A.\ 1)$$

$$\underline{k} \times \hat{\underline{H}} = -\omega\varepsilon \, \hat{\underline{E}} \qquad (A.\ 2)$$

$$\underline{k} \cdot \hat{\underline{E}} = 0 \qquad (A.\ 3)$$

$$\underline{k} \cdot \hat{\underline{H}} = 0 \qquad (A.\ 4)$$

From (1.10), it follows that:

$$\underline{k} = -k \sin \beta_o \cos(\alpha+\phi) \, \underline{i}_x - k \sin \beta_o \sin(\alpha+\phi) \, \underline{i}_y + k \cos \beta_o \underline{i}_z =$$

$$= k_x \, \underline{i}_x + k_y \, \underline{i}_y + k_z \, \underline{i}_z \qquad (A.\ 5)$$

Substituting (A. 5) in (A. 1, 3, 4) we easily get:

$$(k_y^2 + k_z^2) \, \hat{E}_z = \omega\mu \, k_y \, \hat{H}_x - k_x k_z \, \hat{E}_x \qquad (A.\ 6)$$

Similarly, by substituting (A. 5) in (A. 2, 3, 4) we get:

$$(k_y^2 + k_z^2) \, \hat{H}_z = -\omega\varepsilon k_y \, \hat{E}_x - k_x \, k_z \, \hat{H}_x \qquad (A.\ 7)$$

Accordingly:

$$\hat{E}_z(\alpha) = \frac{\cos\alpha\cos\beta\ \hat{E}_x(\alpha) - \sin\alpha\ \zeta\ \hat{H}_x(\alpha)}{1 - \cos^2\alpha\ \sin^2\beta_o} \sin\beta_o \quad (A.\ 8)$$

$$\zeta\hat{H}_z(\alpha) = \frac{\cos\alpha\cos\beta\zeta\hat{H}_x(\alpha) + \sin\alpha\ \hat{E}_x(\alpha)}{1 - \cos^2\alpha\ \ \sin^2\beta_o} \sin\beta_o \quad (A.\ 9)$$

For the y-components we get similarly:

$$\hat{E}_y(\alpha) = -\frac{\sin^2\beta_o\ \sin\alpha\cos\alpha\hat{E}_x(\alpha) + \cos\beta_o\zeta\ \hat{H}_x(\alpha)}{1 - \cos^2\alpha\ \ \sin^2\beta_o} \quad (A.\ 10)$$

$$\zeta\hat{H}_y(\alpha) = -\frac{\sin^2\beta_o\ \sin\alpha\cos\alpha\ \zeta\hat{H}_x(\alpha) - \cos\beta_o\hat{E}_x(\alpha)}{1 - \cos^2\alpha\ \ \sin^2\beta_o} \quad (A.\ 11)$$

APPENDIX B

Evaluation of the constant E_h

From (5. 7) the constant E_h can be put under the form:

$$E_h = \frac{E_{o_x} \cos\phi_o \sin^2\beta_o}{1 - \sin^2\beta_o \cos^2\phi_c} \cdot \left[\sin\phi_o + i \operatorname{cotg}\beta_o R \right] \qquad (B. 1)$$

$$R = \frac{\Psi_h(\alpha_p^+) / \Psi_h(\alpha_p^-) + \Psi_e(\alpha_p^+) / \Psi_e(\alpha_p^-)}{\Psi_h(\alpha_p^+) / \Psi_h(\alpha_p^-) - \Psi_e(\alpha_p^+) / \Psi_e(\alpha_p^-)} \qquad (B. 2)$$

wherein, from (3. 18)

$$\alpha_p^{\pm} = \pm i \ln \operatorname{cotg} \frac{\beta_o}{2} = i \rho \qquad (B. 3)$$

On the other hand, we have from (2. 4)

$$N(\alpha + x) = N(\alpha) + \frac{1}{4\pi} N(\alpha) \frac{2\alpha - \pi \sin\alpha}{\cos \alpha} + \ldots\ldots \qquad (B. 4)$$

Accordingly, for small values of θ ,

$$\frac{\Psi_h(\alpha_p^+)}{\Psi_h(\alpha_p^-)} \simeq \frac{1 + \frac{\sin\beta_o}{2\pi}\left[(\pi + 2i\rho)\theta^+ + (\pi - 2i\rho)\theta^-\right]}{1 + \frac{\sin\beta_o}{2\pi}\left[(\pi - 2i\rho)\theta^+ + (\pi + 2i\rho)\theta^-\right]} \qquad (B. 5)$$

Furthermore

$$N(z) = N(x + iy) = \cos^{1/4} z \exp\left[\frac{1}{2\pi}\int_0^x \frac{t}{\cos t}\,dt\right] \cdot$$

$$\exp\left[\frac{i}{2\pi}\int_0^y \frac{x + it}{\cos(x + it)}\,dt\right] \qquad (B.\ 6)$$

For y >>1,

$$N(x + iy) \quad N(x + i\infty) \qquad (B.\ 7)$$

Accordingly, for θ small and

$$\left[\theta \cot \frac{\beta_o}{2}\right] << 1 \qquad (B.\ 8)$$

we have from (2. 2)

$$\frac{\Psi_e(\alpha_p^+)}{\Psi_e(\alpha_p^-)} \approx 1 \qquad (B.\ 9)$$

In conclusion, we have:

$$R \approx \frac{\pi}{i\rho \sin \beta_o(\theta^+ - \theta^-)} \qquad (B.\ 10)$$

$$E_h \approx \frac{E_{o_x} \cos \phi_o \sin^2 \beta_o}{1 - \sin^2 \beta_o \cos^2 \phi_o} \left[\sin \phi_o + \frac{1}{\theta^+ - \theta^-} \quad \frac{\pi \operatorname{cotg} \beta_o}{\sin \beta_o \ln \operatorname{cotg} \frac{\beta_o}{2}}\right]$$

$$(B.\ 11)$$

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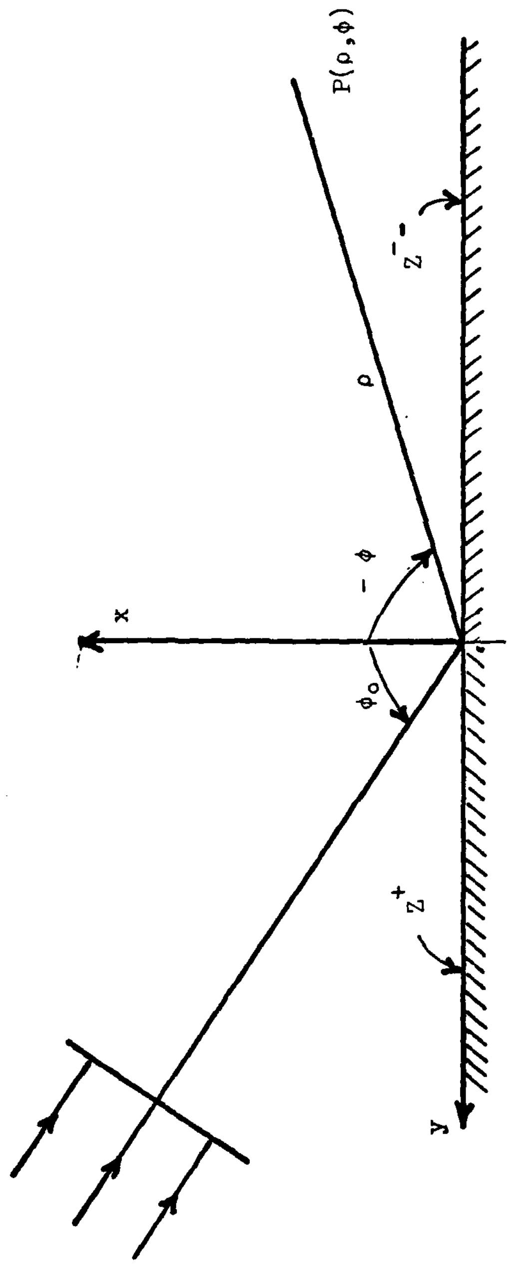

Fig. 1. Geometry of the scattering problem under consideration.
Normal incidence.

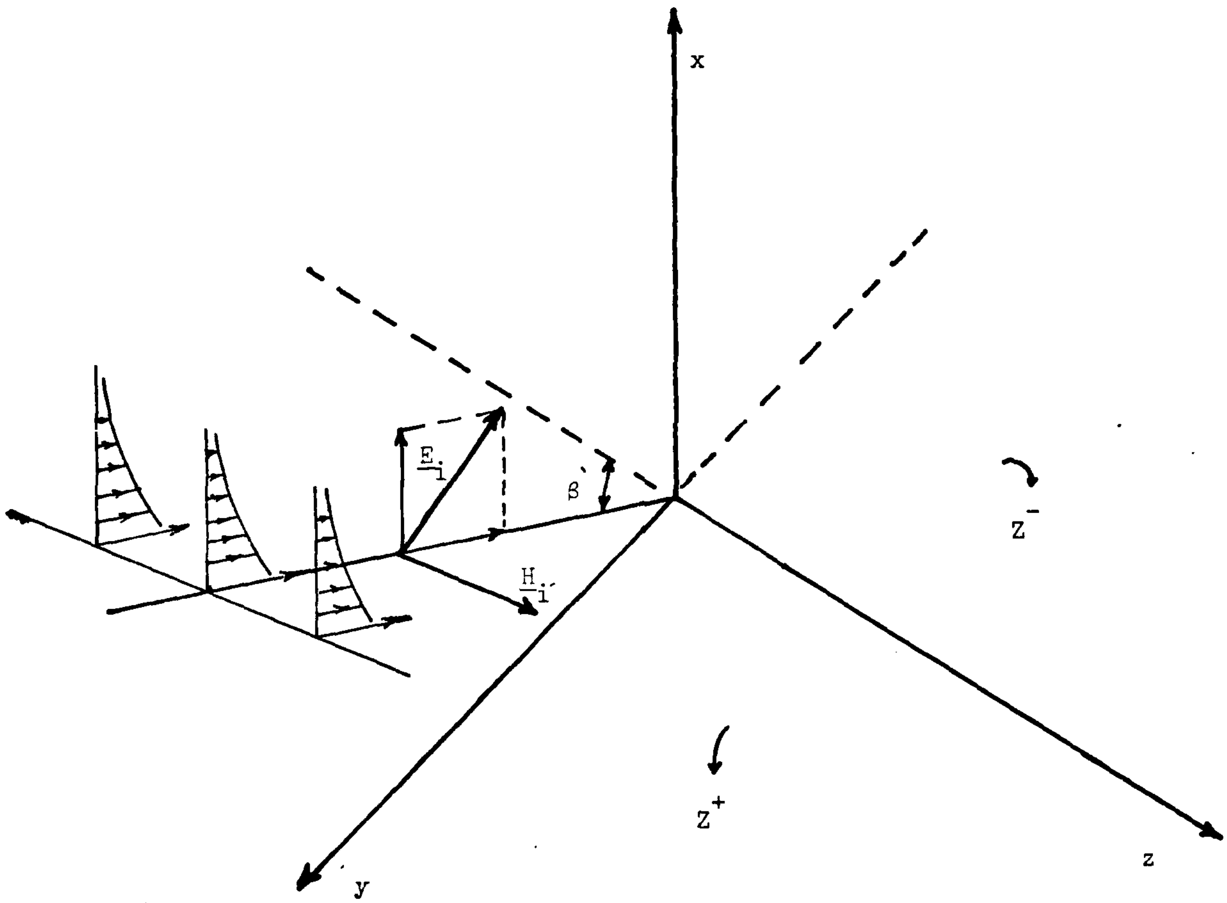

Fig. 2. Geometry of a surface wave obliquely incident on a surface impedance discontinuity

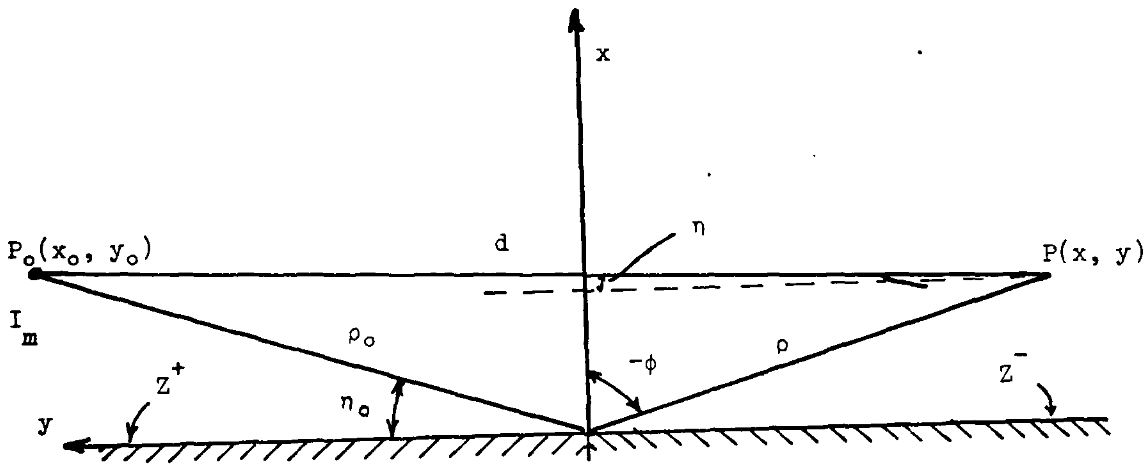

Fig. 3. Geometry of an uniform magnetic source line over a mixed-path Earth.

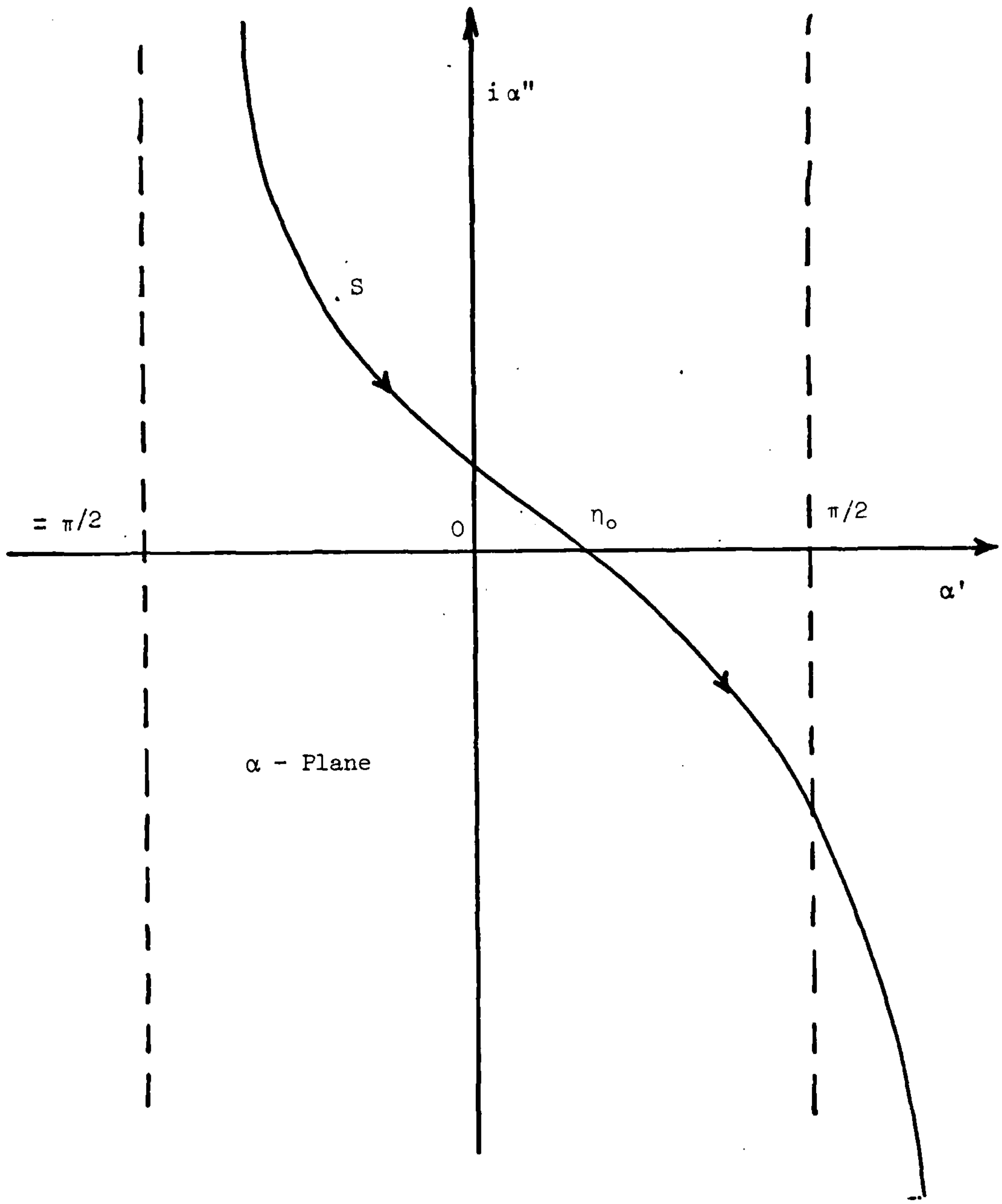

Fig. 4. Sommerfeld contour of integration for the expansion of the Hankel function $H_0^{(1)}(kd)$

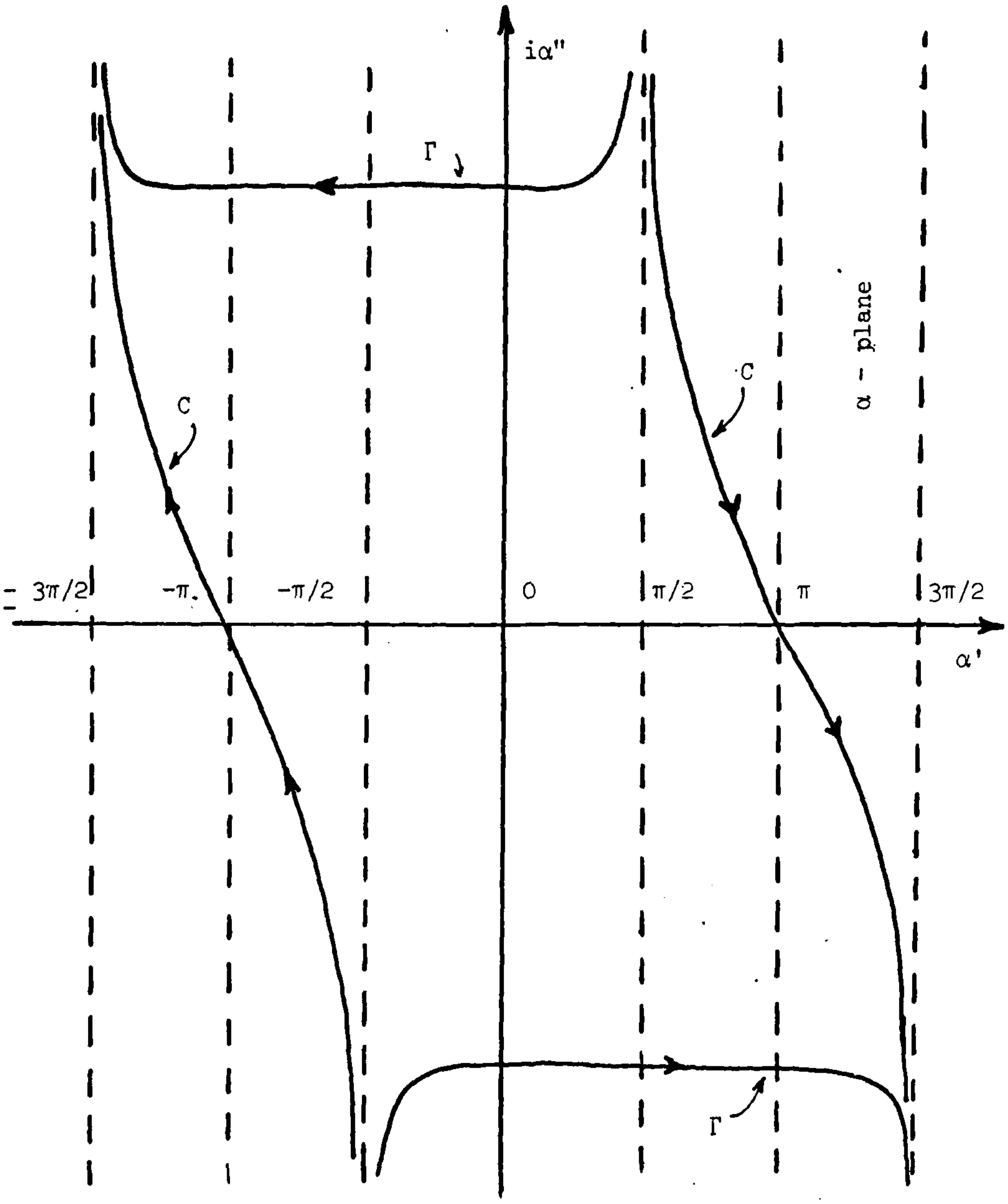

Fig. 5. Integration contour in the compley plane.

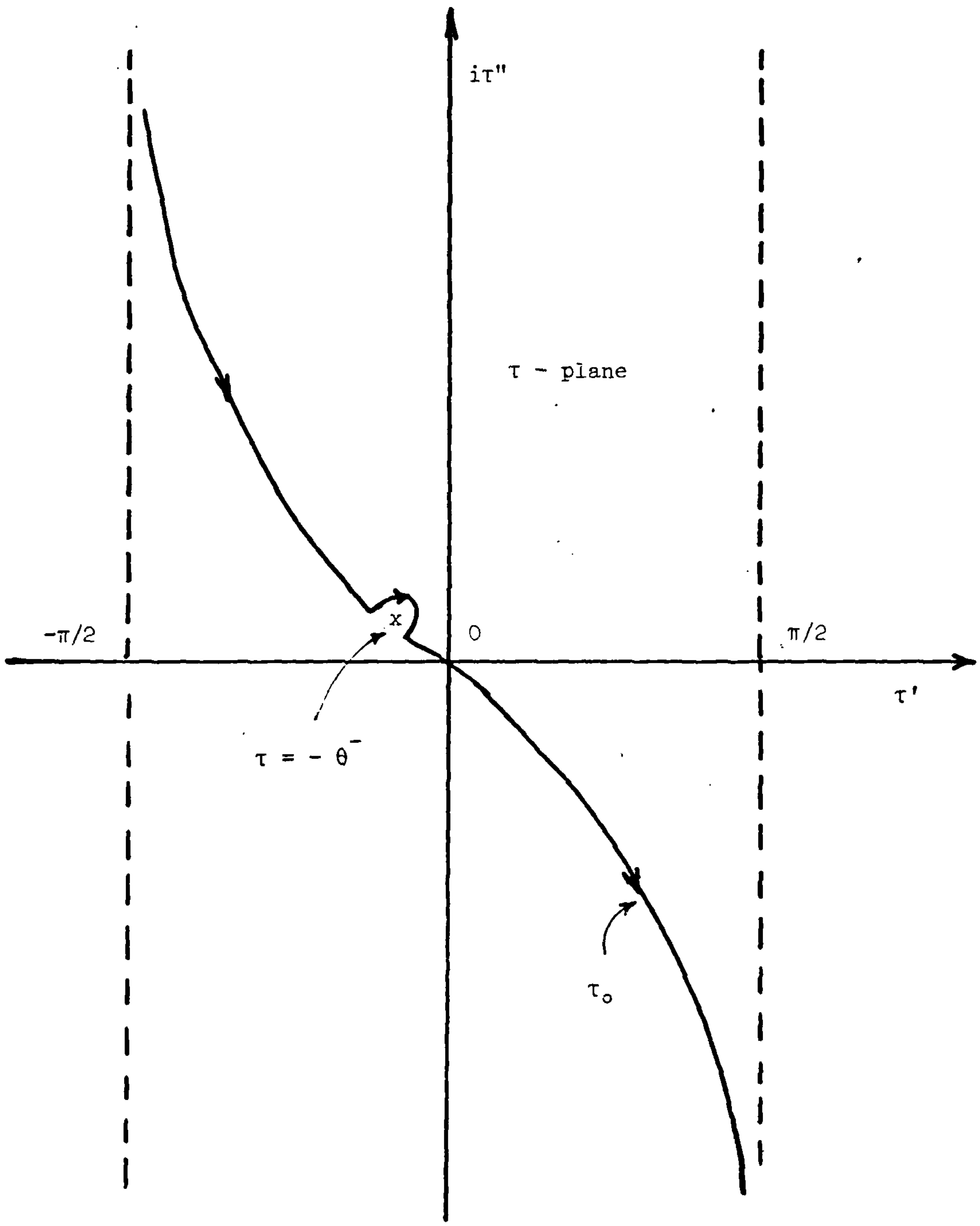

Fig. 6. Integration contour for the trasmitted surface field.

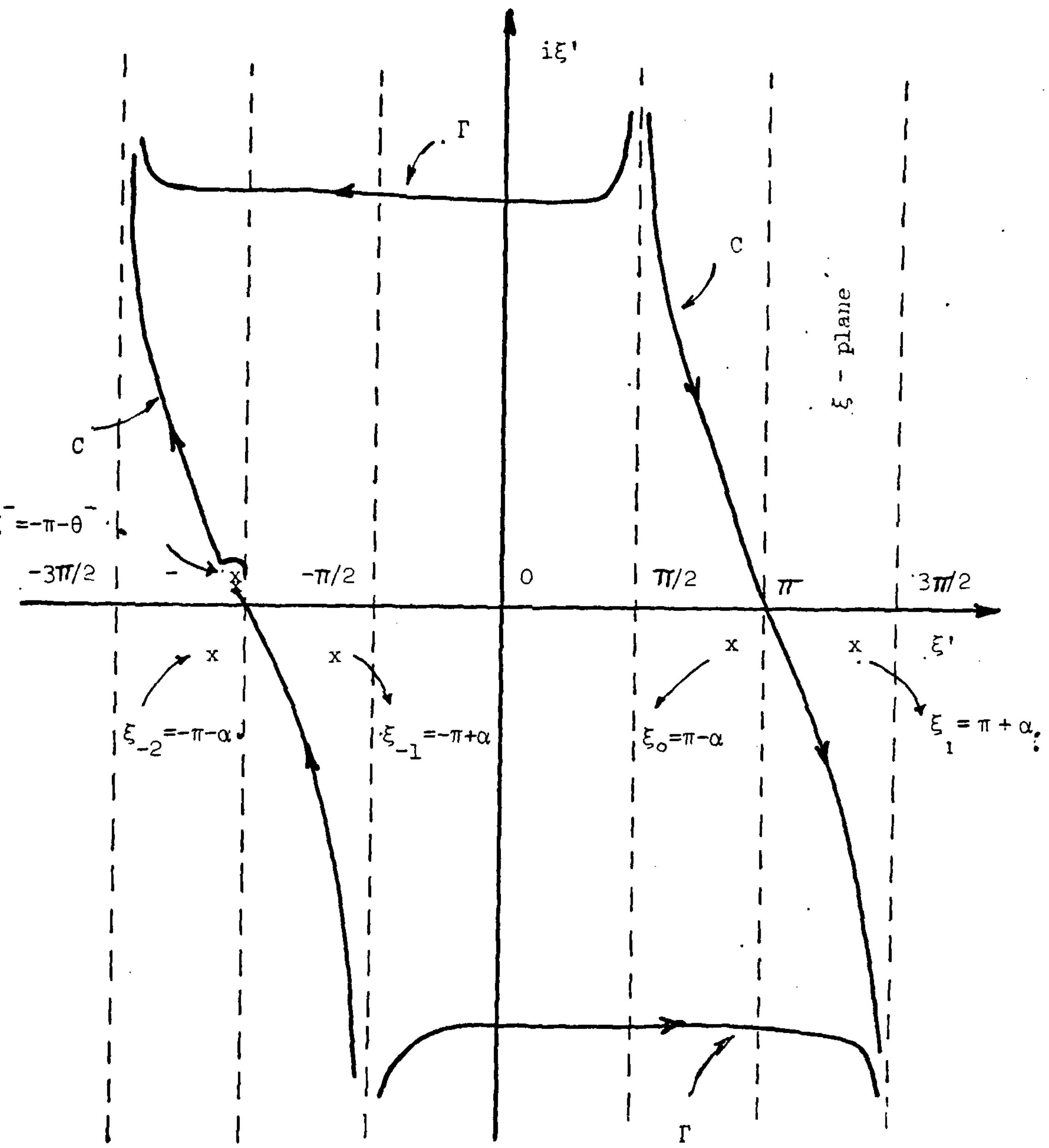

Fig. 7. Integration contour and pole positions for the field produced by a magnetic line source over a mixed-path propagation.

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